String Theory: Making Contact with Hadron Physics

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2001: String Theory revealed as theory of **just about anything**.

But actually, string theory *is* a theory of something.

Why did string theory work at all in context of hadronic physics?

Long history ('t Hooft, Polyakov, many others) led to

Maldacena (1997): **Gauge Theory = Superstring Theory**

Precise conjecture for certain supersymmetric theories.

• A "duality": **nonperturbative** quantum equivalence of classically different theories. (Not related to twistors and **perturbative** calculations.)

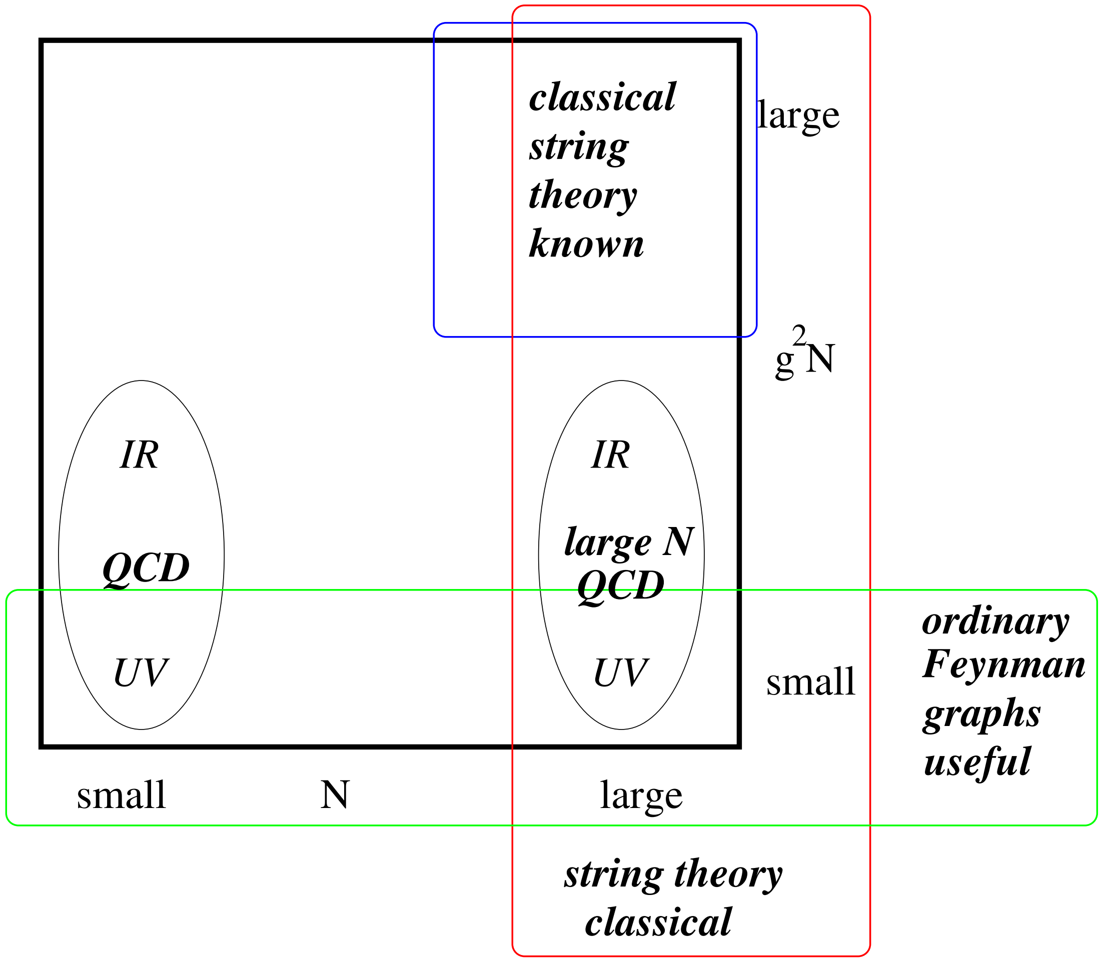
• *Multiple descriptions of one physical theory*

1. Gauge theory in 3+1 dimensions
2. String theory in 3+1+**1** [+5] dimensions

• *Fifth* coordinate "r" is proportional to energy scale μ

• The catch: In any regime, at most one description is simple.

The Other Catch...



[N = number of colors, g = coupling]

So stringy description of real QCD involes a quantum unknown string.

So why care?

I) The $1 \ll g^2 N \ll N$ theories are the best toy models for QCD.

- Exist in 3+1 *Minkowski* dimensions
- Can exhibit infrared confinement and ultraviolet scaling
- Can calculate interesting non-perturbative dynamics not accessible to Feynman diagrams or lattice gauge theory
- Help identify universal/nonuniversal aspects of confining theories
- Useful for supporting/disproving folk theorems, speculations
- May be useful for developing new methodologies

II) These are interesting and natural gauge theories in their own right.

- Might be responsible for electroweak symmetry breaking
- May be observed at LHC – would we know? (Randall-Sundrum)
- Might be responsible for other physics (flavor, supersymmetry breaking, inflation, etc.)

Today: A Case Study

There have been a number of surprisingly successful applications of the *gravity limit* of these toy models to QCD and to pure Yang-Mills theory

- Structure and rigidity of hadronic spectrum, couplings in Yang-Mills, QCD. (Csaki et al. 98;... Erlich et al. 05; Da Rold & Pomerol 05; Sakai & Sugimoto 05)
- Black hole dynamics and RHIC physics: low viscosity fluids and horizons. (Herzog & Son 04; Kovtun et al. 04; ...)
- Quasi-universality of the couplings of the ρ meson. (Hong, Yoon & MJS 04)

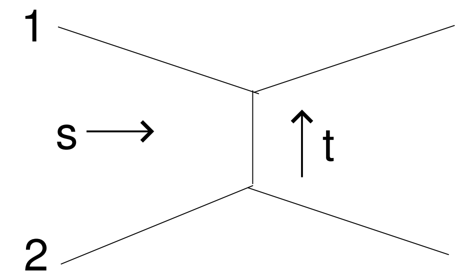
But I will present an application which requires the *stringy physics* of these toy models.

The properties of fast hadrons: BFKL et al.

(Brower, Polchinski, MJS, Tan 05; see also Andreev; Andreev and Seigel; Janik and Peschanski; also Kotikov, Lipatov, Onishchenko & Velizhanin)

When objects are boosted to very high energy, how do they change?

In $2 \to 2$ scattering, t fixed, $|t| \ll s \to \infty$ (large relative boost)



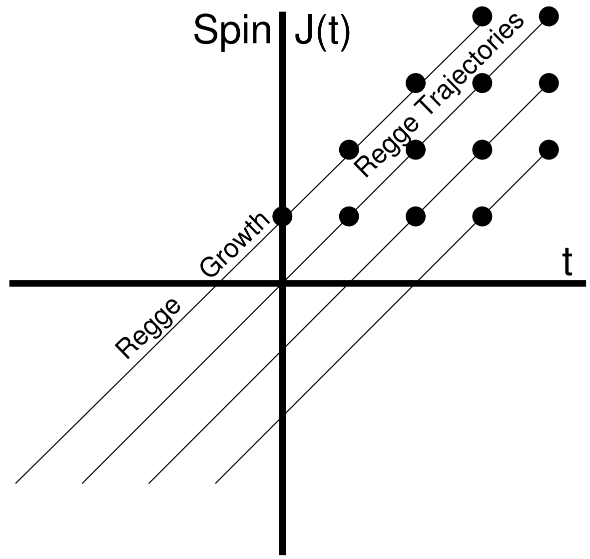
- How do amplitudes grow with energy $\sqrt{s}$?
- How do amplitudes fall with angle ($t < 0$)?
- How do deep inelastic structure functions grow as $x \to 0$?

What happens to strings at large s, fixed t?

Strings in flat space become dense and grow!

String amplitudes $\rightarrow$ Regge behavior $\mathcal{A} \sim s^{J(t)}$

$[J(t) = \alpha(t) = \alpha_0 + \alpha' t]$



- t positive (timelike, unphysical for scattering)
  find massive states with $m^2 = t$ at $J(t) =$ integer

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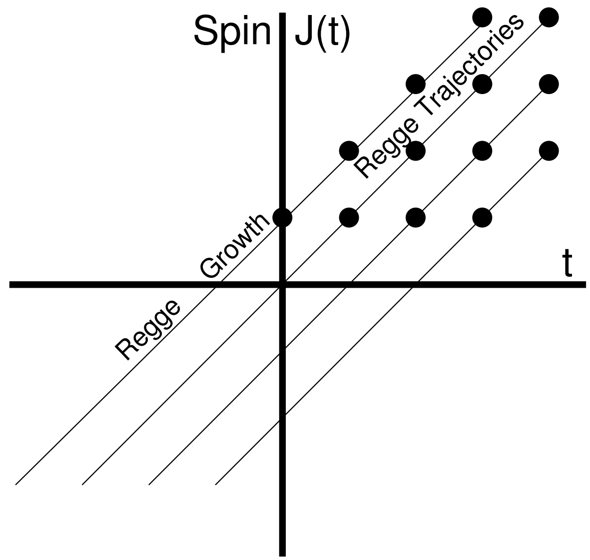

- t negative; Fourier transform momentum space $\longrightarrow$ position space

$$J(t) \sim \alpha_0 + \alpha' t \Rightarrow \mathcal{A} \sim s^{\alpha_0} \; \frac{\exp\left[-|\vec{x}|^2/\alpha' \ln s\right]}{\sqrt{\ln s}}$$

Strings grow in size: $\langle|\vec{x}|^2\rangle \sim \ln s$

[like random-walk diffusion, with time $\ln s$]

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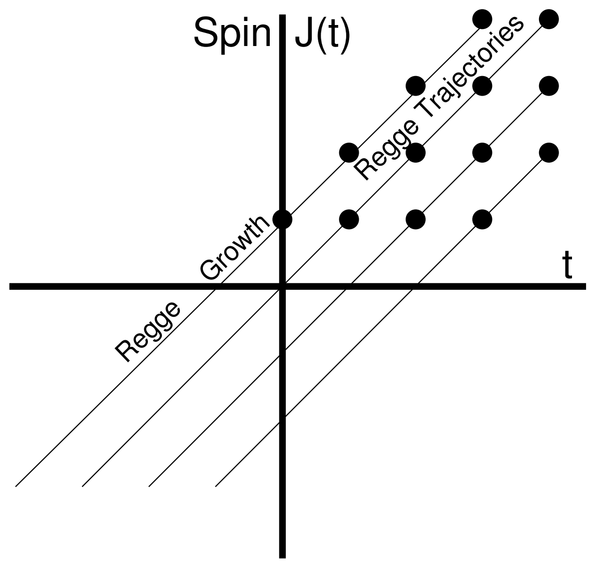
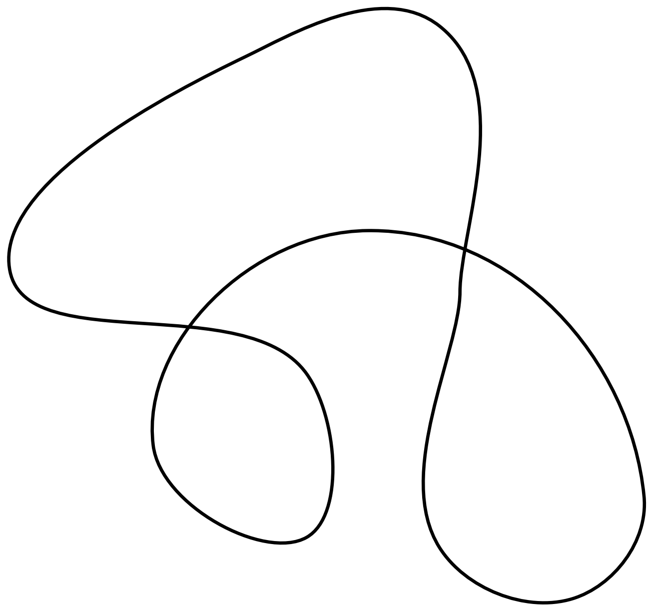


- $t = 0$: (forward scattering)

$$J(0) = \alpha_0 = 2 \;\Rightarrow\; \sigma \sim s$$

Violates unitarity?! Breaks down — Strings become dense, "black"

Diffusion Effect



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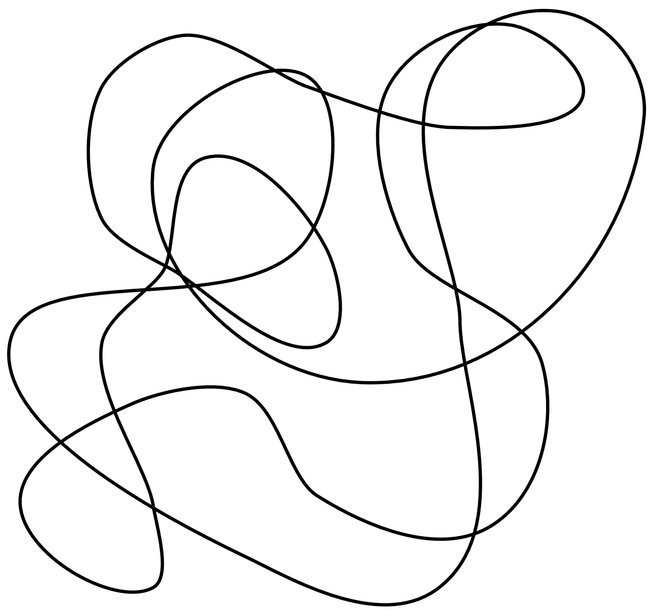

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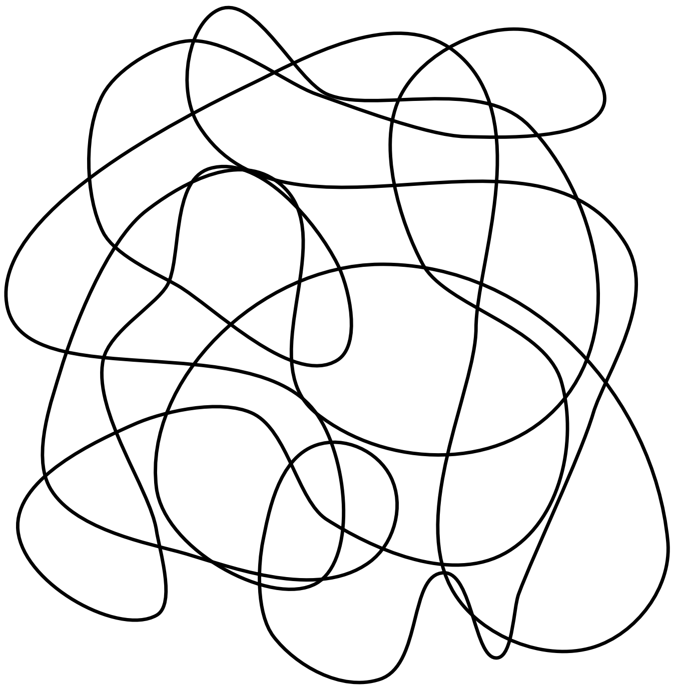

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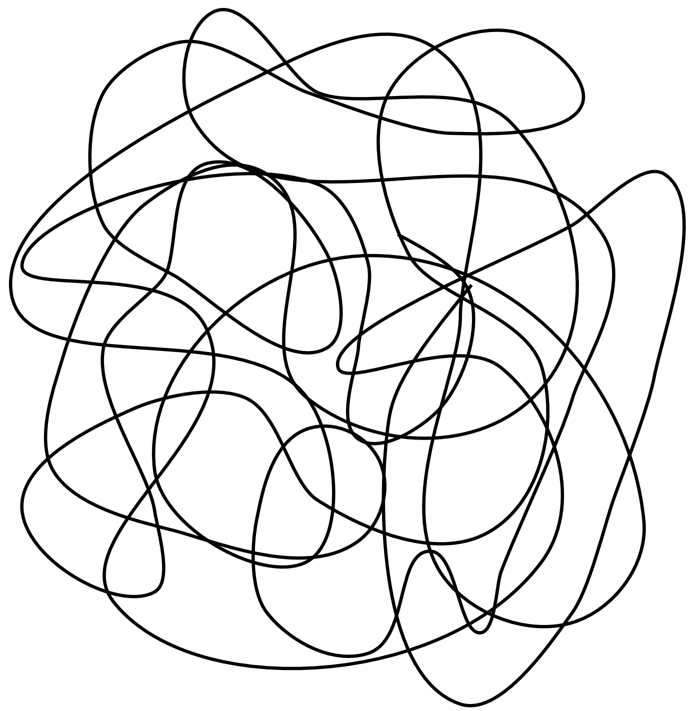

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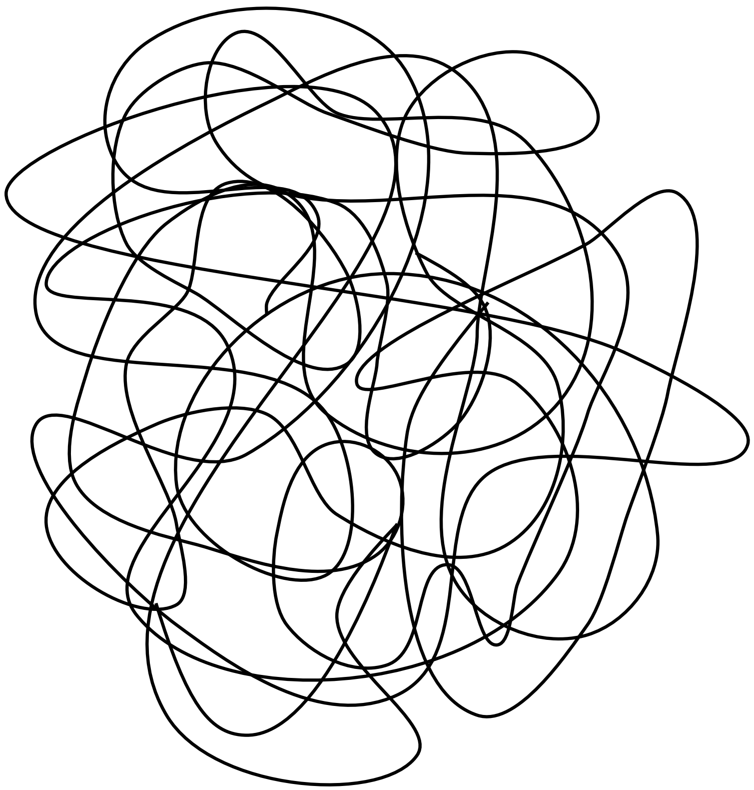

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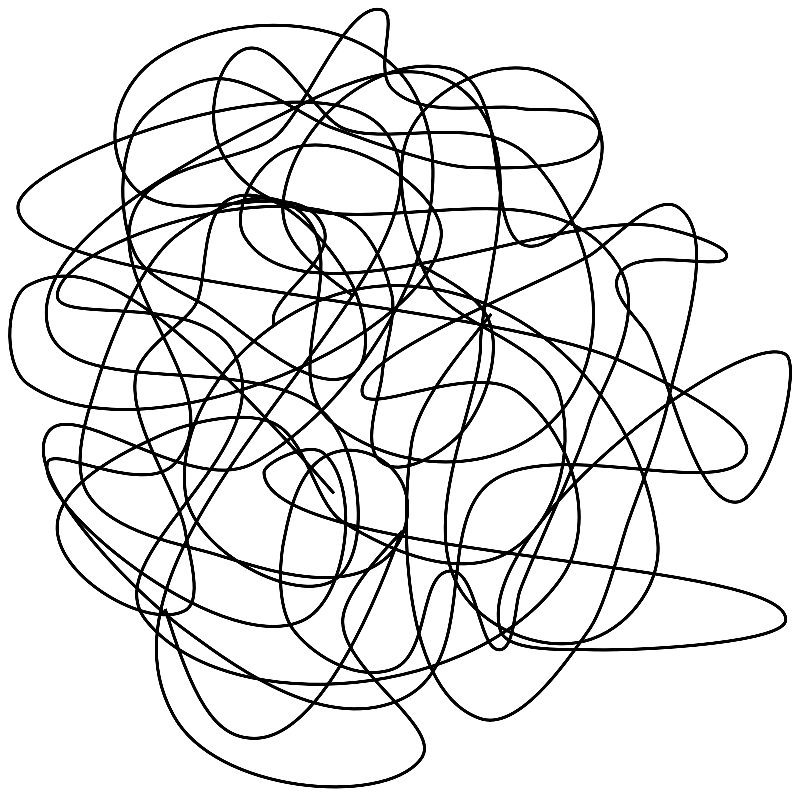
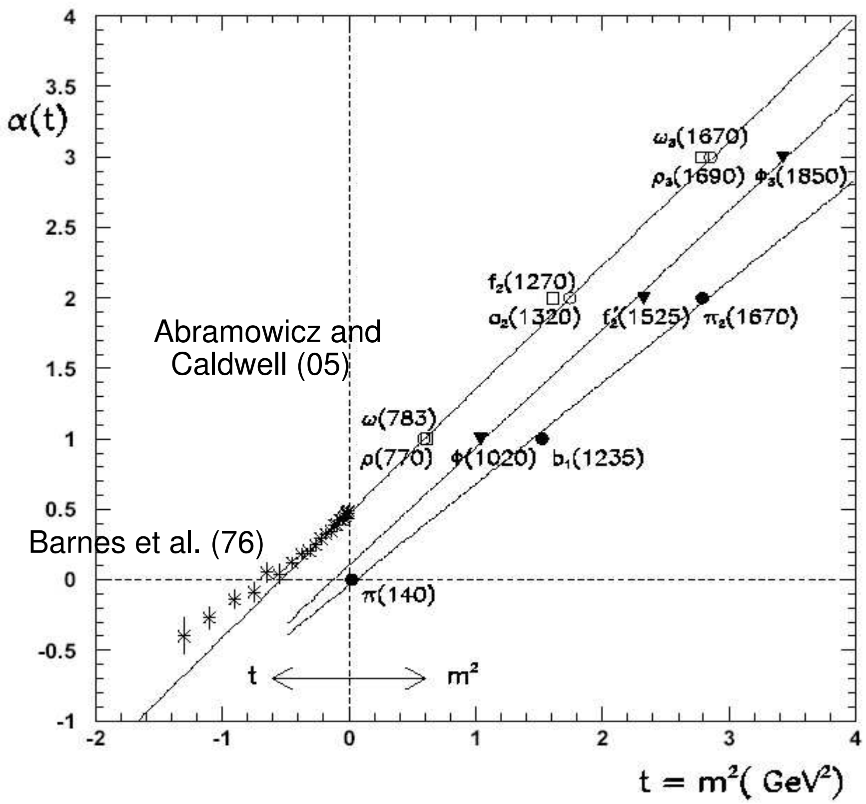


What happens to hadrons at large s, fixed t?

What's true for strings is true for hadrons: (ρ channel)



For $t > -1$ GeV^2, hadrons are just like strings in flat space.

- Their masses lie on linear trajectories

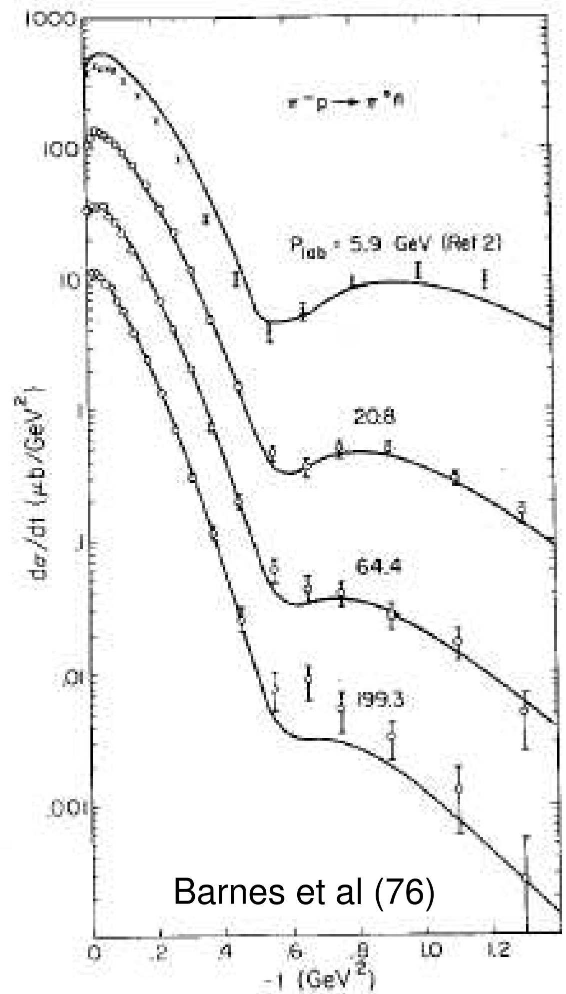


Fig. 1

Shows Gaussian falloff $d\sigma/dt \sim s^{-\alpha'|t|}$

- They grow; due to “wee partons” at small x.

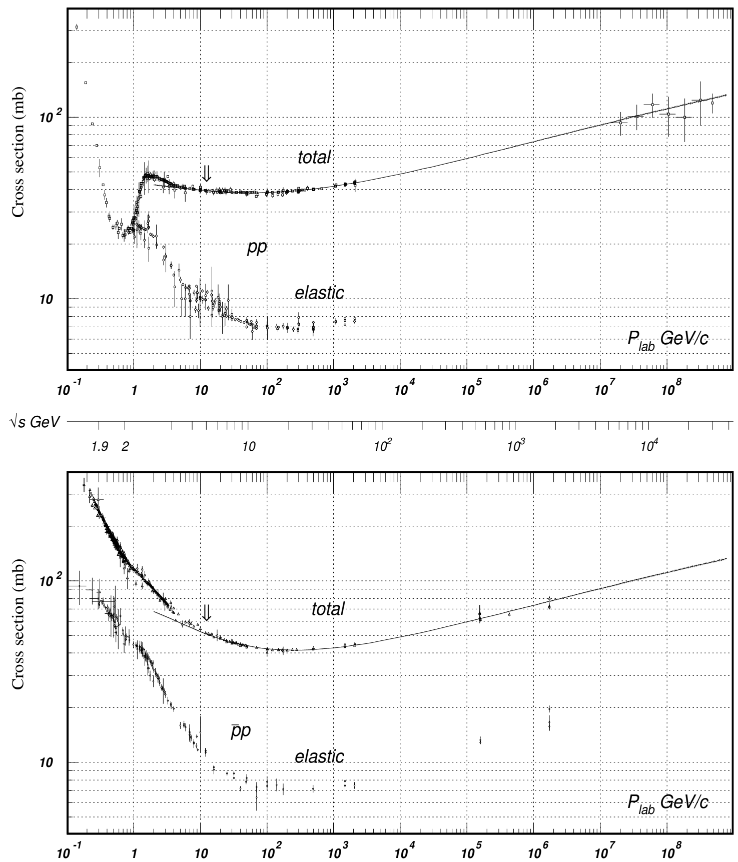
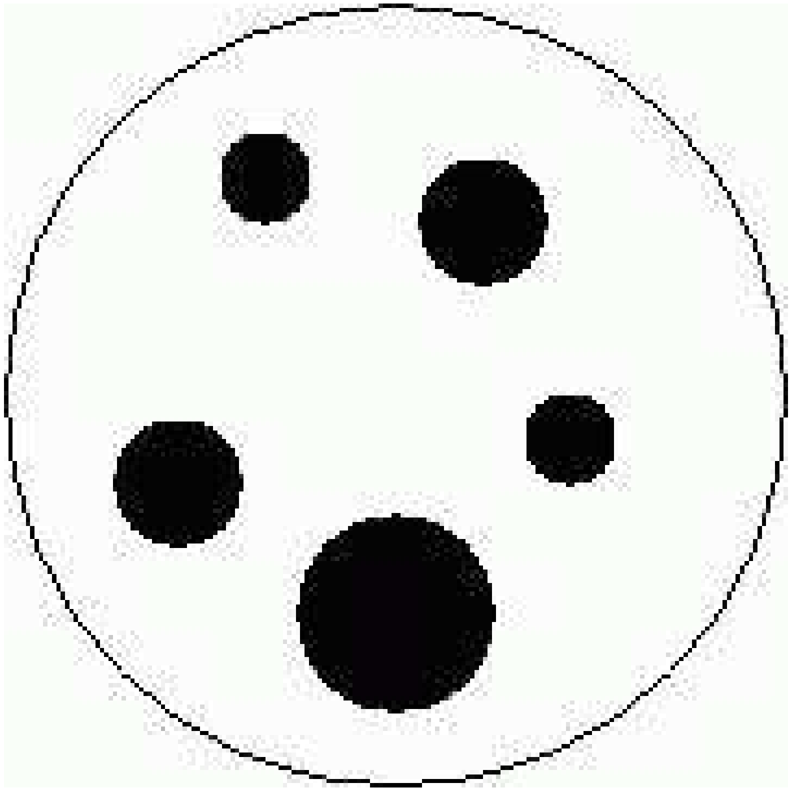


Figure 40.11: Total and elastic cross sections for pp and $\bar{p}p$ collisions as a function of laboratory beam momentum and total center-of-mass energy. Corresponding computer-readable data files may be found at http://pdg.lbl.gov/xsect/contents.html (Courtesy of the COMPAS group, IHEP, Protvino, August 2003)

- They become dense; growing cross-sections that threaten unitarity.

Diffusion Effect



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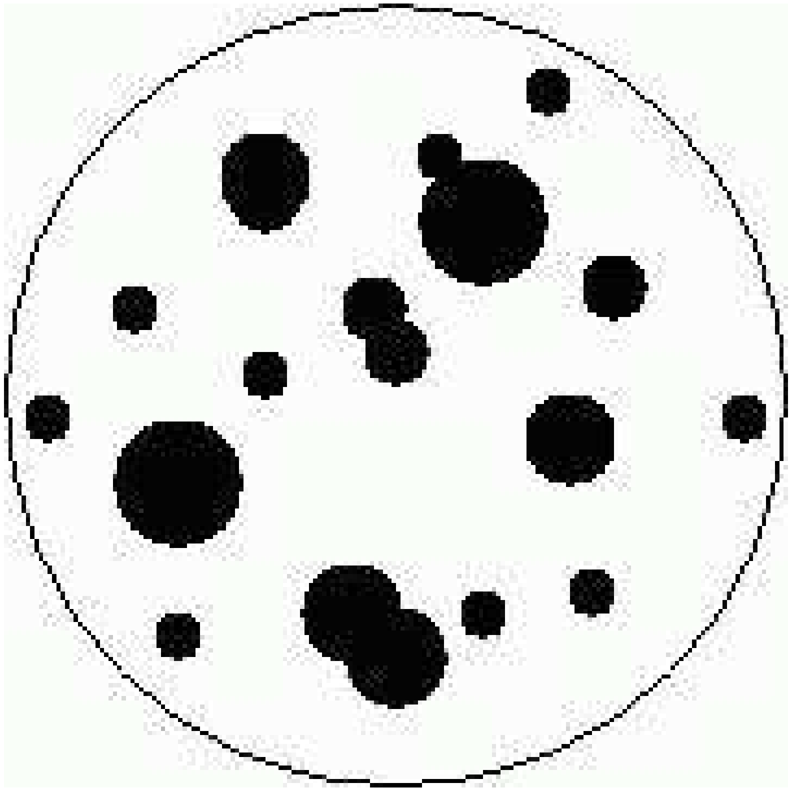

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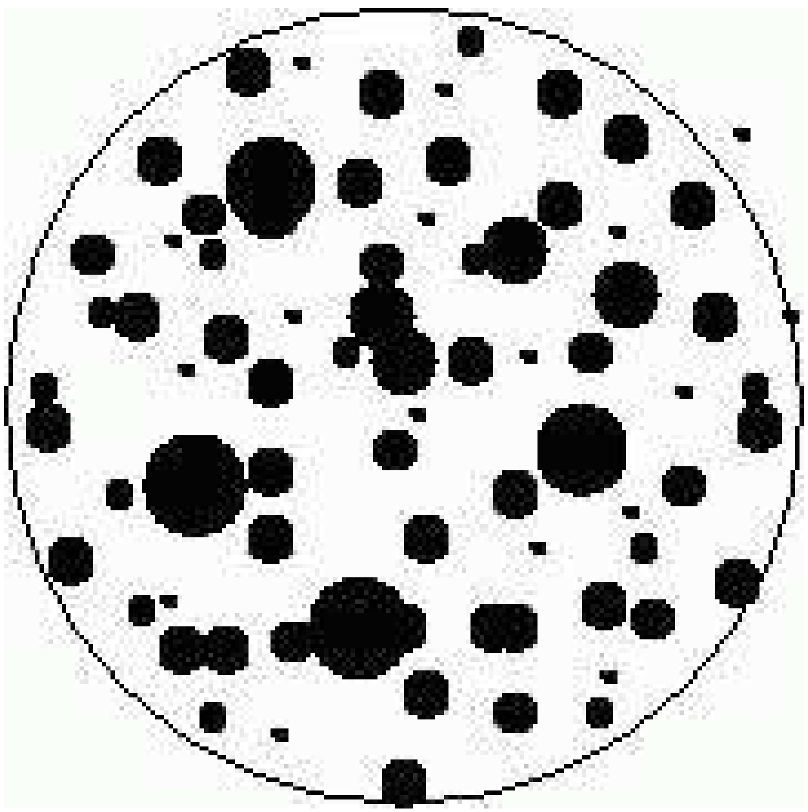

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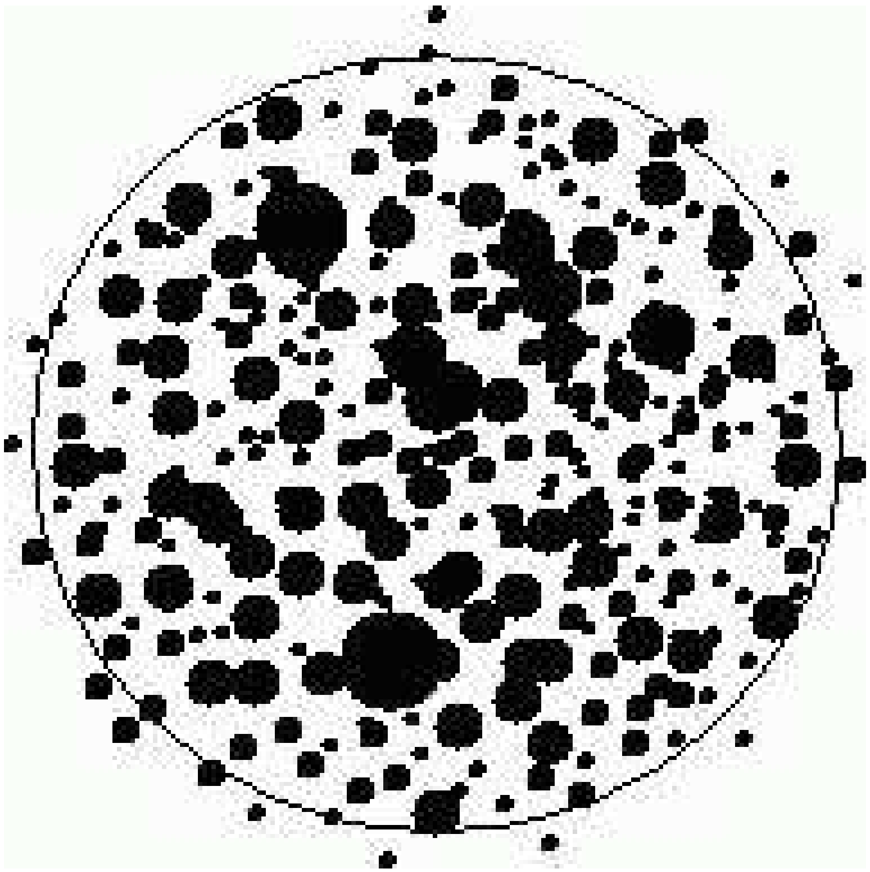

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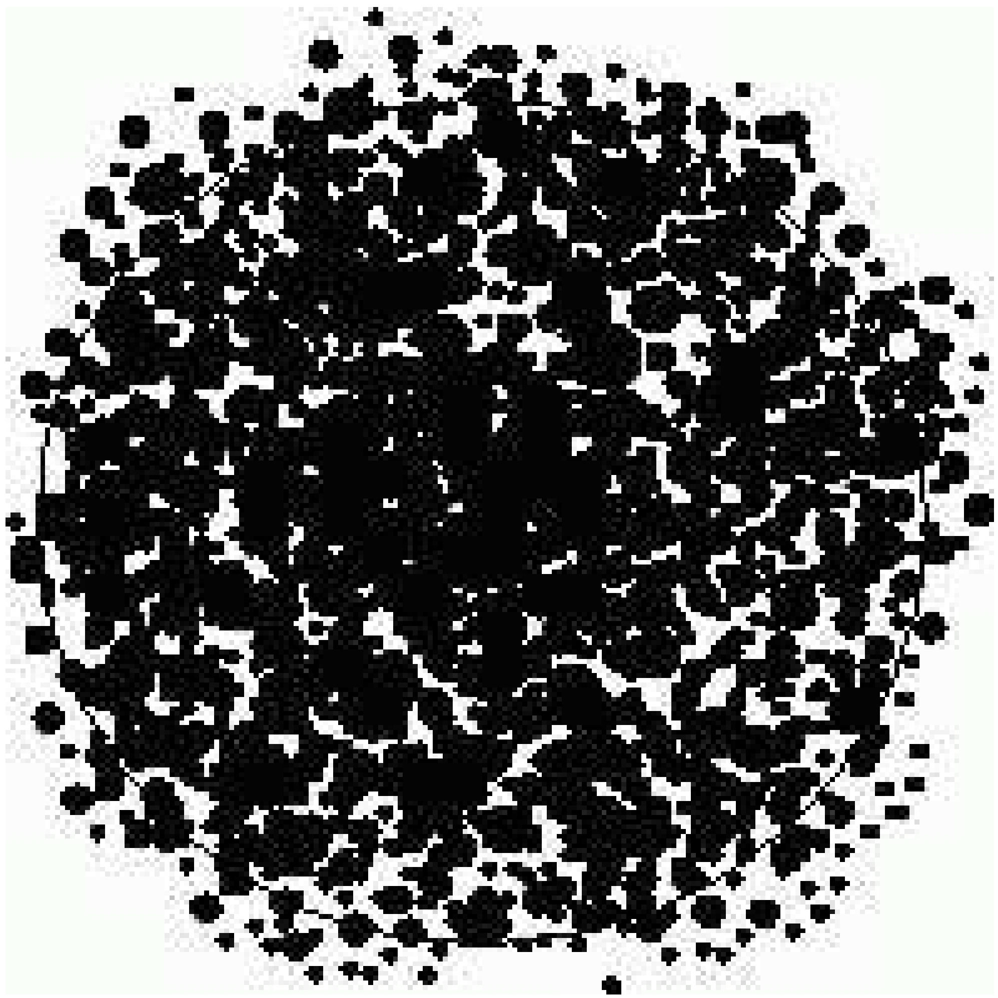

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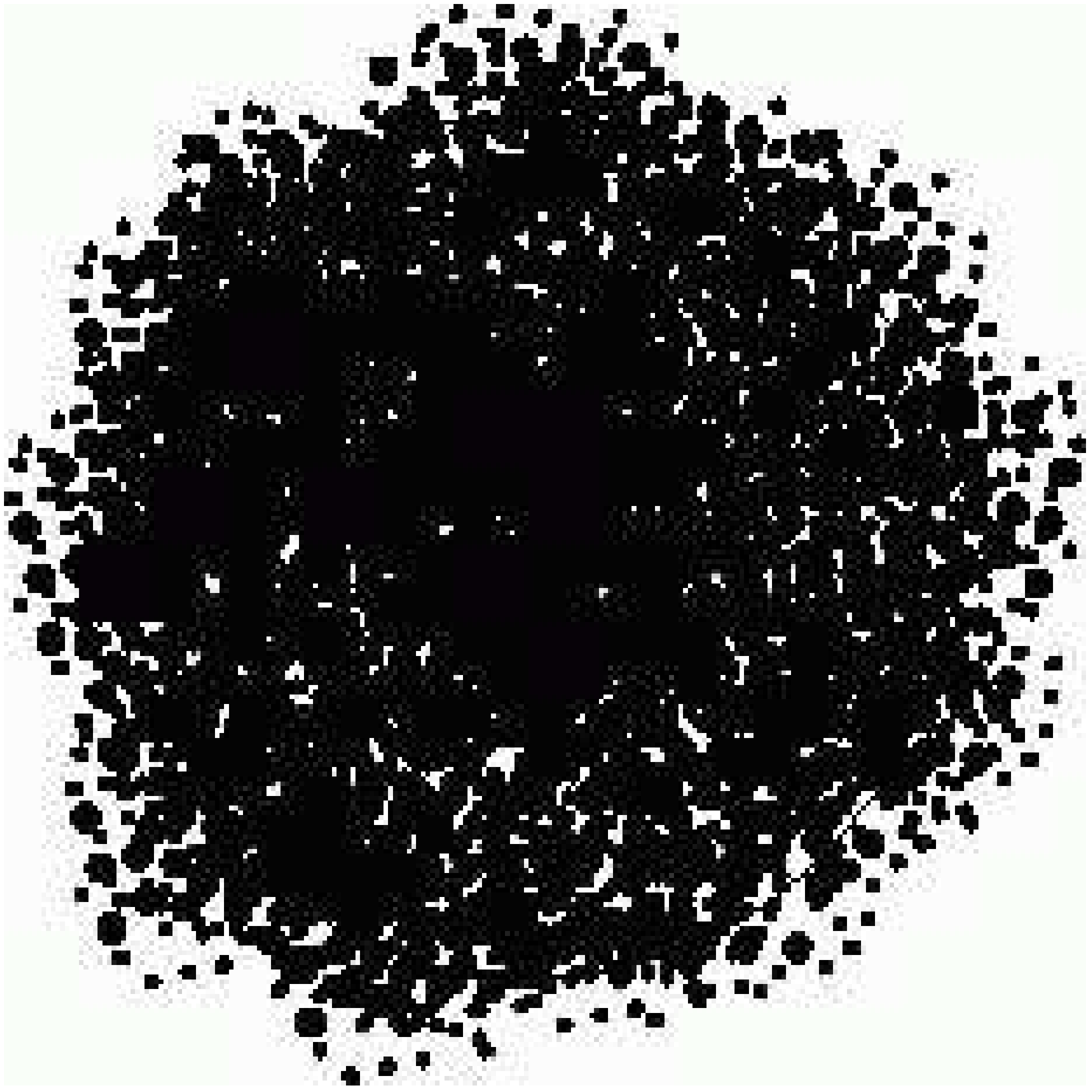


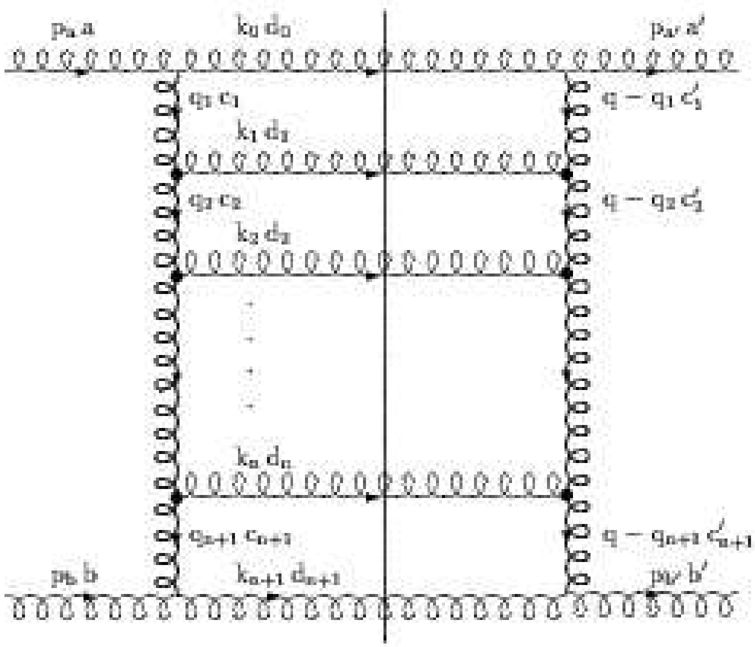
- Can this behavior be understood or calculated?
  - Not perturbative
  - Lorentzian character
- Is this behavior universal to all gauge theories?
- What happens when $t \ll -1$ GeV^2, where strings and hadrons differ?

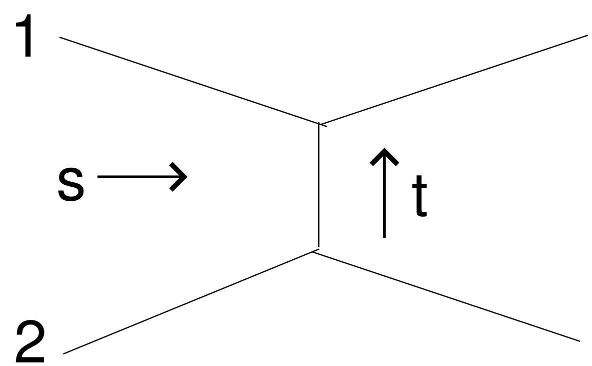
BFKL: Balitsky & Lipatov; Fadin, Kuraev & Lipatov 75
attempt to solve the relevant problem of amplitudes at large s by resumming perturbation theory in analogy with RG

Fix t, then

Sum all $(\alpha_s \ln s)^n$ *terms at leading order in* α_s.

BFKL Resummation





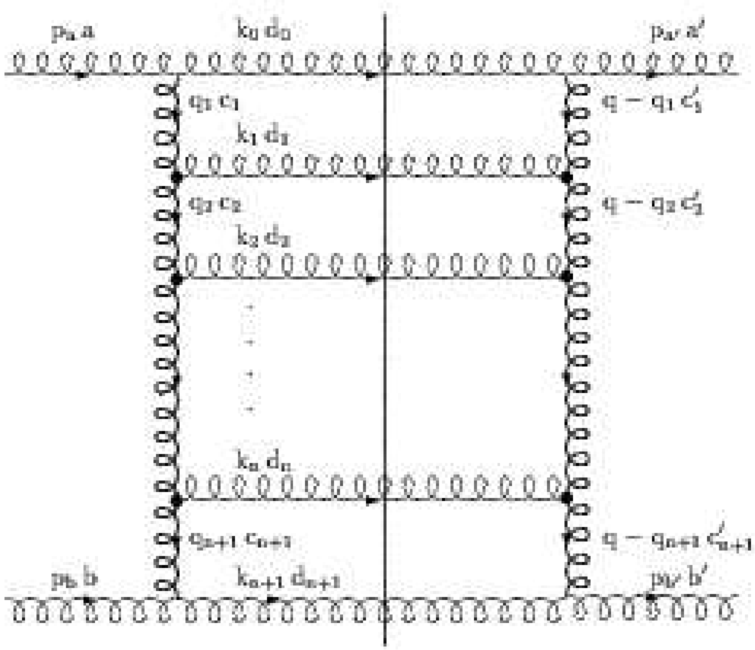
- In the regime $|t| \ll s$ (large energy, small scattering angle) the object exchanged is not a single hadronic resonance but a coherent combination of colorless physical objects called the "Pomeron".

- "BFKL kernel" is equivalent to *Pomeron propagator*

- Simplest to compare BFKL and string at $N \gg 1$, constant–α_s, $t = 0$

- QCD: $N = 3$, running–α_s, and BFKL most reliable at $t \ll -\Lambda^2$; more on this later.

BFKL Resummation



BFKL Kernel $K(s, k_\perp, k'_\perp)$

Here $k_\perp, k'_\perp$ are transverse momenta flowing through top, bottom line

Resum leading $\ln s$ terms using ladders of ladders of ladders of ladders of...

BFKL at $t = 0$

$$\mathcal{A}_{2\to 2} = \int \frac{dk_\perp}{k_\perp} \int \frac{dk'_\perp}{k'_\perp} \Phi_1(k_\perp) K(s; k_\perp, k'_\perp) \Phi_2(k'_\perp)$$

After much work, obtain power of s times a *diffusion kernel* with space $= \ln k_\perp$, time $= \ln s$

$$K(s, k_\perp, k'_\perp) \approx s^{j_0} \frac{e^{-\left[(\ln[k'_\perp/k_\perp])^2/4\mathcal{D}\ln s\right]}}{\sqrt{\pi \ln s}}$$

where

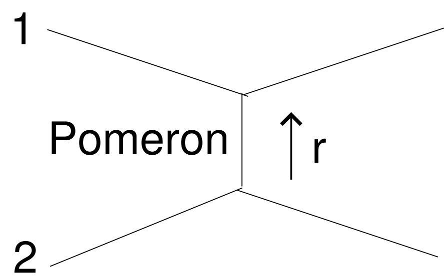
$$j_0 = 1 + \frac{4\ln 2}{\pi}\alpha N \ , \quad \mathcal{D} = \frac{14\zeta(3)}{\pi}\alpha N \ .$$

Enter String Theory

At very small constant α_s, the BFKL calculation is *universal*: independent of N, matter content, etc.

So can recalculate it in a theory with a string description, large N, adjustable $g^2 N$

- At small $g^2 N$, compute kernel from BFKL resummation — infinite set of Feynman diagrams.
- At large $g^2 N$, compute kernel using string theory — single tree-level $2 \to 2$ string diagram, calculated on curved $3 + 1 + \mathbf{1}$ $[+5]$ dimensional space.



$$
\begin{aligned}
\mathcal{A} \sim s^{J(t)} &= s^{2+\alpha' t/2} \quad \text{(flat space)} \\
&\to s^{2+\alpha' \nabla^2/2} \quad \text{(curved space)} \\
&= s^2 e^{(\alpha' \ln s)\nabla^2/2} \equiv s^2 e^{-H\tau}
\end{aligned}
$$

where $\tau \propto \ln s$ is again a diffusion time, and

$$H \propto -\nabla^2 = -\frac{1}{r^2}\nabla_{3+1} - \nabla_{\mathbf{r}}^{\mathbf{2}} + 0 = -\partial_u^2 + (4 - e^{-2u}t/t_0) = -\partial_u^2 + V(u;t)$$

where $u = \ln r$ and the effective potential $V(u;t) = 4 - [t/t_0]e^{-2u}$

So for $t = 0$, $V(u;0) = 4$,

$$\mathcal{A} \to s^{j_0} e^{-\#\tau[-\partial_u^2]} \quad , \quad j_0 = 2 - \frac{2}{\sqrt{g^2 N}} \tag{1}$$

$$\mathcal{A} \sim s^{j_0} e^{-\# \tau [-\partial_u^2]}$$

Sandwiching this operator between the two scattering hadrons, and writing the kernel explicitly

$$\mathcal{A} \sim \int du \int du' \; \Phi_1(u) s^{j_0} \frac{e^{-\left[(u-u')^2/4\mathcal{D}\tau\right]}}{\sqrt{\pi\tau}} \Phi_2(u')$$

where $j_0 = 2 - \frac{2}{\sqrt{g^2 N}}$ and $\mathcal{D} = \frac{1}{\sqrt{g^2 N}}$, $\tau \propto \ln s$, $u = \ln r$

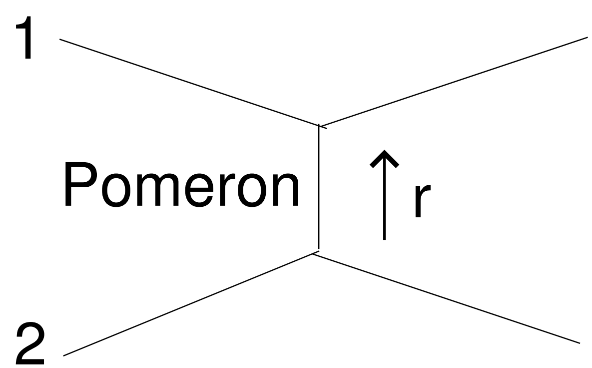
Compare this result to the BFKL kernel for $t = 0$

$$\mathcal{A} = \int \frac{dk_\perp}{k_\perp} \int \frac{dk'_\perp}{k'_\perp} \Phi_1(k_\perp) s^{j_0} \frac{e^{-\left[(\ln[k'_\perp/k_\perp])^2/4\mathcal{D}\ln s\right]}}{\sqrt{\pi \ln s}} \Phi_2(k'_\perp)$$

$$j_0 = 1 + \frac{4\ln 2}{\pi}\alpha N \; , \quad \mathcal{D} = \frac{14\zeta(3)}{\pi}\alpha N \; .$$

Same form, with $r \to k_\perp$, different coefficients $j_0, \mathcal{D}$.

In this tree-level string calculation, the exchanged Pomeron — the graviton trajectory — is propagating in the curved 5th dimension!



- BFKL result is just Regge behavior in 5 curved dimensions
- Amplitude exactly of BFKL *form*, with $k_\perp \to r$.
- BFKL diffusion is Regge diffusion with space $= \ln r$, time $= \ln s$.
- Coefficients differ (as expected; $g^2 N$ is different)
- Form of the answer could have been predicted in advance; conformal invariance.

What about $t < 0$?

- Took 8 years to extend BFKL to $t < 0$
- Very easy in string theory; simply requires studying spectrum of differential operator $H = -\partial_u^2 + V(u; t)$.

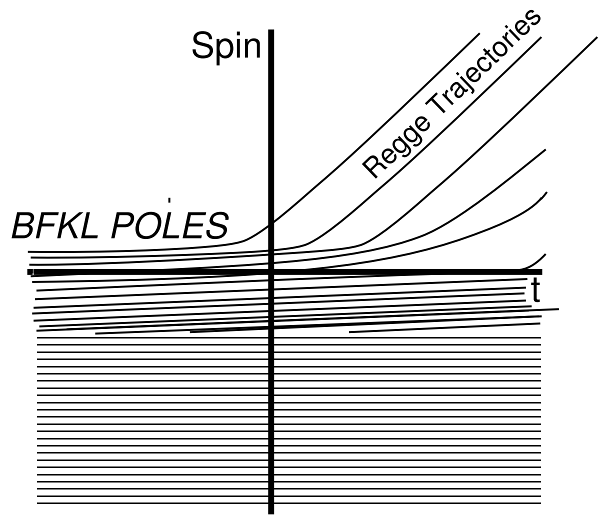
Confinement and hadrons at $t > 0$?

- Again, simply requires studying spectrum of differential operator $H = -\partial_u^2 + V(u; t)$ with appropriate boundary conditions.

We can obtain the single-Pomeron exchange amplitude *at all* t: the BFKL behavior, the hadronic resonances, and the transition between them.

Logarithmically-running coupling?

- In QCD
  - one loop correction is large, only reliable at large $-t$ (small α_s)
  - BFKL "cut" turns into BFKL "poles"
- In string theory
  - Running simply alters effective potential $V(u; t)$
  - Regge trajectories at positive t evolve smoothly in t to BFKL "poles"



$2 \rightarrow 2$ Amplitude via Single Pomeron-Exchange

Note: for small N, multi-Pomeron effects may change small t region.

We conclude

• BFKL approach is sensible and criticisms of its general structure are too strong

• But near $t = 0$ (even for $N \gg 1$) we find the confinement- and hadron-independent kernel need not dominate amplitudes; no evidence kernel can predict small-x deep inelastic scattering, no matter how large is Q^2.

Note: Consistent with failure of BFKL kernel to directly explain the small-x data at HERA

(though there are already perfectly good arguments in this regard concerning the connection between DGLAP evolution and BFKL, and we do not claim to contradict these *e.g* Ciafaloni et al.)

• Old speculations that Regge trajectories $J(t)$ asymptote to negative integers as $t \to \infty$ are disproven.

Summary

Large–g^2N gauge theories are coming under increasing theoretical control using supergravity and superstring theory.

They provide

- toy models for and alternative viewpoints on QCD
- new model-building possibilities

The first applications of this formalism to issues of general theoretical importance in QCD are currently appearing.

• We find the form of the BFKL result is reproduced in string theory, from Regge behavior in curved ten-dimensional space.

• Our (large-N) result extends to both positive and negative t and shows how the BFKL kernel connects to the Regge trajectories at $t > 0$; note real QCD may have important multi-Pomeron effects near $t = 0$.

• We find BFKL kernel unlikely to predict small-x behavior in deep inelastic scattering.

For the Future

The string theory calculation of BFKL kernel in large $g^2 N$ regime was easy — much easier than small-$g^2 N$ calculation in gauge theory.

Unfortunately applicability to experiment is limited, and the problem currently is of mostly formal interest.

- *Are there other challenging BFKL-like questions for which there is no existing formalism, but for which the string theory calculation is still relatively easy?*
- *Are there other pressing questions for which the string theory calculations are much harder but for which the payoff is much greater?*

e.g. Jets at LHC? [What's the question?!]